# Sets Approach Algorithms for Determination of Component Minimal Complete Subgraph

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**Abstract:** Giving a graph G we wish to determine the component minimal complete subgraph of G. For this goal we present two algorithms in terms of sets representation of graph G. We give too the theorems that characterized this algorithms.

## Paper sumary

1. General definitions and results
2. Algorithm without vertexes and edges elimination
3. Algorithm with vertexes and edges elimination
4. Conclusions and supplementary results

References

# General definitions and results

**Definition 1.7** Let G=(V,E) be a graph. If for any $x \in V$ there exist $y,z \in V$ , $y \neq z$ so that $\{\{x,y\},\{x,z\},\{y,z\}\} \subset E$ we say that G is a ***minimal complete graph***.

**Denition 1.9** Let G=(V,E) be a graph. If any component in G is a minimal complete graph then G is named ***component minimal complete graph***.

Let $CMC_G$ designate the component minimal complete subgraph for the graph G, if such a subgraph exists.

# General definitions and results

**Proposition 1.2.** Let $G=(V,E)$ be a graph and $X=\{x\in V|\omega(x) = 0\}$ with $V-X\neq\emptyset$. Let consider the subgraph $H=(V-X,F)$ of G. Then, G has a component minimal complete subgraph if and only if H has a component minimal complete subgraph. In addition, we have $CMC_G = CMC_H$.

**Proposition 1.3.** Let $G=(V,E)$ be a graph and $X=\{x\in V|\omega(x) = 1\}$. Let consider the subgraph $H=(V-X,F)$ of G. Then, G has a component minimal complete subgraph if and only if H has a component minimal complete subgraph. In addition, we have $CMC_G = CMC_H$.

**Proposition 1.4.** Let $G=(V,E)$ be a graph and let $CMC_G=(U,F)$ be the component minimal complete subgraph for G. Then for any $x\in U$ in G we have $\omega(x)\geq 2$.

# Algorithm without vertexes and edges elimination

Let G=(V,E) be a grap k=|E| and so, $E=\{m_1,m_2,\ldots,m_k\}$.

The Algorithm 1:

Step 1. Let $U=\varnothing$, $F=\varnothing$ and $i=1$.

Step 2. If $i=k-2$ then STOP otherwise continue.

Step 3. Let $j=i+1$.

Step 4. If $j=k-1$ then $i\leftarrow i+1$ and go to Step 2, otherwise continue.

Step 5. If $m_i\cap m_j=\varnothing$ then go to Step 9, otherwise continue.

Step 6. Let $p=(m_i\cup m_j)-(m_i\cap m_j)$.

Step 7. If $p\notin\{m_{j+1},\ldots,m_k\}$ then go to Step 9, otherwise continue.

Step 8. Let $U\leftarrow U\cup m_i\cup m_j$ and $F\leftarrow F\cup\{m_i,m_j,p\}$.

Step 9. Let $j\leftarrow j+1$ and go to Step 4.

# Algorithm without vertexes and edges elimination

**Theorem 2.1.** Let G=(V,E) be a graph with |E|=k. Then by applying the algorithm 1 we obtain $CMC_G$ and the algorithm has the complexity $O(k^3)$.

# Algorithm with vertexes and edges elimination

Step 1. Let $Y=\{x\in X|\omega(x)=0\}\cup\{x\in X|\omega(x)=1\}$.
Step 2. If $Y=\varnothing$ then go to Step 4, otherwise continue.
Step 3. Let $X\leftarrow X-Y$, $T\leftarrow T-\{m\in T|m\cap Y\neq\varnothing\}$ and go to Step 1.
Step 4. Let $V=X$, $E=T$, $k=|E|$ and the graph $G=(V,E)$ for which we consider that $E=\{m_1,m_2,\dots,m_k\}$.
Step 5. Let $U=\varnothing$, $F=\varnothing$ and $i=1$.
Step 6. If $i=k-2$ then STOP otherwise continue.
Step 7. Let $j=i+1$.
Step 8. If $j=k-1$ then $i\leftarrow i+1$ and go to Step 6, otherwise continue.
Step 9. If $mi\cap mj=\varnothing$ then go to Step 13, otherwise continue.
Step 10. Let $p=(m_i\cup m_j)-(m_i\cap m_j)$.
Step 11. If $p\notin\{m_{j+1},\dots,m_k\}$ then go to Step 13, otherwise continue.
Step 12. Let $U\leftarrow U\cup m_i\cup m_j$ and $F\leftarrow F\cup\{m_i,m_j,p\}$.
Step 13. Let $j\leftarrow j+1$ and go to Step 8.

# Algorithm with vertexes and edges elimination

**Theorem 3.1.** Let H=(X,T) be a graph. Then by applying the algorithm 2 we obtain $CMC_H$ and the algorithm has the complexity $O(k^3)$.

# Conclusions and supplementary results

Obtaining component minimal complete subgraph can be interesting, but someone could be interested in effectively finding all subgraphs of a given graph that are complete graph with three vertexes.

The goal can be easily reached just with minor modication of the algorithms indicated in section 2 and 3.

We consider that it is interesting to study what it is happened if we can apply the algorithms to disconnected graphs so that the determination of CMC works separately on every component of a given graph. I wish to do this task in a future paper. Also I wish to give in a next paper similar algorithm for algebraic approach.

# Important reference

Bârza, S., *Some Consideration about Component Minimal Complete Subgraph*, Analele Universitatii Spiru Haret, Seria Matematica-Informatica, anul VII, nr. 2, Editura Fundatiei România de Mâine, Bucuresti 2012.