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On some intuitionistic fuzzy models

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Abstract

- Intuitionistic-fuzzy sets and numbers are considered to show their applicability to optimize the component-based system reliability, in multi-criteria decision-making algorithms, and the assessment of learning level using intuitionistic-fuzzy automata.
- The described results were published, by separate papers, previously, and only the power of intuitionistic-fuzzy modelling is emphasized here.

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Intuitionistic Fuzzy Sets and Numbers

- A fuzzy set (firstly introduced by Zadeh(1965)) is called a fuzzy number if its membership function increases monotonously to a unique maximum degree equal to 1 and then decreases monotonously.
- In the case of intuitionistic-fuzzy numbers, both a membership and a non-membership function are given.
- In fuzzy computing with intuitionistic fuzzy numbers, the most used membership (and non-membership) functions have triangular or trapezoidal shape.
- When the universe of discourse X is a non empty and finite set, a vague set A of the universe of discourse U can be represented by a true-membership function tA and a false-membership function fA. A vague number is a vague subset in the universe of discourse X that is both normal (the maximum value of the true membership function is 1) and convex (similar to fuzzy convex sets). Vague calculus uses triangular vague sets, trapezoidal vague sets, and general vague sets (Kumar et al, 2008).

Intuitionistic-Fuzzy Numbers - TIFN

If A is a TIFN (Triangular Intuitionistic Fuzzy Number) then A is described by five real numbers

$$(a_1, a_2, a_3; a', a''),\ \ a' \le a_1 \le a_2 \le a_3 \le a'',$$

and two triangular functions. The membership function and the non-membership function are given by

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}, \qquad \nu_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a'}, & \text{for } a' \le x \le a_2 \\ \dfrac{x - a_2}{a'' - a_2}, & \text{for } a_2 \le x \le a'' \\ 1, & \text{otherwise} \end{cases}.$$

Intuitionistic-Fuzzy Numbers - TrIFN

Let $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$. A trapezoidal Intuitionistic Fuzzy Number A in $\boldsymbol{R}$ (TrIFN), written as

$$(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4'),$$

has the following membership and non-membership functions:

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, \text{ for } a_1 \le x \le a_2 \\ 1, \text{ for } a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, \text{ for } a_3 \le x \le a_4 \\ 0, \text{ otherwise} \end{cases}, \qquad \nu_A(x) = \begin{cases} \dfrac{a_2-x}{a_2-a_1'}, \text{ for } a_1' \le x \le a_2 \\ 0, \text{ for } a_2 \le x \le a_3 \\ \dfrac{x-a_3}{a_4'-a_3}, \text{ for } a_3 \le x \le a_4' \\ 1, \text{ otherwise} \end{cases}.$$

TIFN properties

- If TIFN $A = (a_1, a_2, a_3; a', a'')$, and $k > 0$, then the TIFN kA is given by $(ka_1, ka_2, ka_3; ka', ka'')$.
- If TIFN $A = (a_1, a_2, a_3; a', a'')$, and $k < 0$, then the TIFN kA is given by $(ka_3, ka_2, ka_1; ka'', ka')$.
- If $A = (a_1, a_2, a_3; a', a'')$ and $B = (b_1, b_2, b_3; b', b'')$ are TIFNs, then the sequence defined by $(a_1+b_1, a_2+b_2, a_3+b_3; a'+ b', a''+b'')$ describes the TIFN $A \oplus B$;
- If $A = (a_1, a_2, a_3; a', a'')$ and $B = (b_1, b_2, b_3; b', b'')$ are TIFNs, then the TIFN $A \otimes B$ is described by $(a_1b_1, a_2b_2, a_3b_3; a'b', a''b'')$.

Intuitionistic Fuzzy Reliability Computation

Let S be a software system integrating a number of components according to a serial architecture. This is the case of single processor computer systems running the software during a period of time T. For simplicity reason we assume that every component is called only once, but this is not a serious constraint. If R_j is the intuitionistic fuzzy reliability of the j^{th} component, and R_S is the intuitionistic fuzzy reliability of the entire system (with n items), and $R_j = (r_{j1}, r_{j2}, r_{j3}; r_j', r_j'')$, then $R_S = R_1 \otimes R_2 \otimes \cdots \otimes R_n$, defined by $(r_1, r_2, r_3; r', r'')$, with: $r_i = \prod_{j=1}^{n} r_{ji}$, i = 1, 2, 3, $r' = \prod_{j=1}^{n} r_j'$, and $r'' = \prod_{j=1}^{n} r_j''$

If S is a software system composed by n items running in parallel, using the above notations, we evaluate the intuitionistic fuzzy reliability of S by: $R_S = 1 \ominus \prod_{j=1}^{n} (1 \ominus R_j)$, defined by $(r_1, r_2, r_3; r', r'')$, with: $r_i = 1 - \prod_{j=1}^{n} (1 - r_{ji})$, $i = 1, 2, 3$, $r' = 1 - \prod_{j=1}^{n} (1 - r_j')$, and $r'' = 1 - \prod_{j=1}^{n} (1 - r_j'')$.

Using the above methodology intuitionistic fuzzy reliability formulas can be derived for hybrid software architectures.

Intuitionistic Numbers in Reliability Optimization – 1st step

Let be the optimization problem in the most general form ($m = n+1$, the m^{th} constraint is related to the cost-budget inequality):

$$\begin{cases} \max f(\lambda) \\ g_j(\lambda) \leq 0 \\ \mathrm{j}=1, 2, \ldots, \mathrm{m}. \end{cases}$$

Step 1.

Using a Monte-Carlo approach, an approximate lower bound (minimum) L^a and an approximate upper bound (maximum) U^a can be obtained. According to the intuitionistic fuzzy principle, the degree of non-membership (rejection) and degree of membership (acceptance) are considered so that the sum of both values is less than one.

Let L^r (resp. U^r) be the lower (resp. upper) bound of the objective function such that $L^a \leq L^r \leq U^r \leq U^a$.

It is known that "for objective function of maximization problem, the upper bound for non-membership function is always less than that of the upper bound of membership functions."

It is possible, for a given θ (decision maker choice) to take $U^r = U^a - \theta$, and $L^r = L^a$.

Intuitionistic Fuzzy Optimization Algorithm – Step 2

Let be the optimization problem in the most general form ($m = n+1$, the m^{th} constraint is related to the cost-budget inequality):

$$\begin{cases} \max \ f(\lambda) \\ g_j(\lambda) \le 0 \\ j = 1, 2, \ldots, m. \end{cases}$$

Step 2.

Define the membership and non-membership functions for the uncertain objective function. Various models can be used. A simple proposal is:

$$\mu\ (f(\lambda)) = \begin{cases} 0, \text{ for } f(\lambda) \le L^a \\ \dfrac{f(\lambda) - L^a}{U^a - L^a}, \text{ for } L^a \le f(\lambda) \le U^a \\ 1, \ f(\lambda) \ge U^a \end{cases} \quad \text{and} \quad \nu(f(\lambda)) = \begin{cases} 1, \text{ for } \lambda \le L^r \\ \dfrac{U^r - f(\lambda)}{U^r - L^r}, \text{ for } L^r \le f(\lambda) \le U^r \\ 0, \text{ otherwise.} \end{cases}$$

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Intuitionistic Fuzzy Optimization Algorithm – Step 3

Let be the optimization problem in the most general form ($m = n+1$, the m^{th} constraint is related to the cost-budget inequality):

$$\begin{cases} \max f(\lambda) \\ g_j(\lambda) \le 0 \\ j = 1, 2, \ldots, m. \end{cases}$$

Step 3.

Define the membership and non-membership functions for the constraint equations. Let j be the constraint index (the model (2)), ξ_j - the tolerance for the degree of constraint acceptance, and ψ_j – the tolerance for the degree of constraint rejection ($\psi_j = \rho\xi_j$, where $0 < \rho < 1$). A proposal for the member-ship and non-membership functions for the constraint with index j follows ($j = 1, 2, \ldots, m$):

$$\mu_j(g_j(\lambda)) = \begin{cases} 1, \text{ for } g_j(\lambda) \le 0 \\ \dfrac{\xi_j - g_j(\lambda)}{\xi_j}, \text{ for } 0 \le g_j(\lambda) \le \xi_j \\ 0, g_j(\lambda) \ge \xi_j \end{cases} \quad \text{and} \quad \nu_j(g_j(\lambda)) = \begin{cases} 0, \text{ for } g_j(\lambda) \le \xi_j - \psi_j \\ \dfrac{g_j(\lambda) - (\xi_j - \psi_j)}{\psi_j}, \text{ for } \xi_j - \psi_j \le g_j(\lambda) \le \psi_j \\ 1, \text{ otherwise.} \end{cases}$$

Intuitionistic Fuzzy Optimization Algorithm – Step 4

Let be the optimization problem in the most general form ($m = n+1$, the m^{th} constraint is related to the cost-budget inequality):

$$\begin{cases} \max f(\lambda) \\ g_j(\lambda) \le 0 \\ \mathrm{j = 1, 2, \ldots, m.} \end{cases}$$

Step 4.

The intuitionistic fuzzy version of the above model is a multi-objective optimization problem:

Maximize $\{\mu(f(\lambda)), \mu_1(g_1(\lambda)), \mu_2(g_2(\lambda)), \ldots, \mu_m(g_m(\lambda))\}$

Minimize $\{\nu(f(\lambda)), \nu_1(g_1(\lambda)), \nu_2(g_2(\lambda)), \ldots, \nu_m(g_m(\lambda))\}$

Subject to

$$\begin{aligned}
&\mu(f(\lambda)) + \nu(f(\lambda)) \le 1; \\
&\mu_j(g_j(\lambda)) + \nu_j(g_j(\lambda)) \le 1, j=1, 2, \ldots, m; \\
&\mu(f(\lambda)) \ge \nu(f(\lambda)); \\
&\mu_j(g_j(\lambda)) \ge \nu_j(g_j(\lambda)), j=1, 2, \ldots, m; \\
&\nu(f(\lambda)) \ge 0; \\
&\nu_j(g_j(\lambda)) \ge 0, j=1, 2, \ldots, m; \\
&\lambda \in \Lambda.
\end{aligned}$$

Intuitionistic Fuzzy Optimization Algorithm – Steps 5 & 6

Step 5.

Reduce the above problem to a bi-objective optimization programming problem, like in the model (based on equal importance assumption):

Maximize $\mu_S(\lambda)=\mu(f(\lambda))+\mu_1(g_1(\lambda))+ \ldots +\mu_m(g_m(\lambda))$

Minimize $\nu_S(\lambda)=\nu(f(\lambda))+\nu_1(g_1(\lambda))+ \ldots +\nu_m(g_m(\lambda))$

Subject to

$$\mu(f(\lambda))+\nu(f(\lambda)) \le 1;\ \mu_j(g_j(\lambda))+\nu_j(g_j(\lambda)) \le 1,\ j=1, 2, \ldots, m;$$
$$\mu(f(\lambda)) \ge \nu(f(\lambda));\ \mu_j(g_j(\lambda)) \ge \nu_j(g_j(\lambda)),\ j=1, 2, \ldots, m;$$
$$\nu(f(\lambda)) \ge 0;\quad \nu_j(g_j(\lambda)) \ge 0,\ j=1, 2, \ldots, m;$$
$$\lambda \in \Lambda.$$

If unequal importance case is considered then some weights (degree of importance) and weighted sums will be used when modeling $\mu_S(\lambda)$ and $\nu_S(\lambda)$.

Step 6.

Finally, use a global optimization method (including Monte Carlo) to find the solution.

Intuitionistic Fuzzy Human Learning -1

- The aim of Bloom's model (Anderson et al, 2001) is to classify the levels of intellectual behaviour in learning. Three overlapping domains are considered by Bloom: cognitive, psychomotor, and affective.
- The paper addresses only the cognitive domain which benefits of six cognitive levels (described by nouns): **Knowledge** (remembering of previously learned material), **Comprehension** (understanding the meaning of material), **Application** (using the learned material in new situations), **Analysis** (ability to apply specific methods on material), **Synthesis** (restructuring the material to obtain new patterns or structures), and **Evaluation** (by judging the value of material).

Intuitionistic Fuzzy Human Learning -2

- The model was revised by Anderson to reflect better the actions (described by verbs):

a - **Remembering** (recognizing and recalling),

b - **Understanding** (by constructing meaning from instructional messages, including oral, written, and graphic communication),

c - **Applying** (executing and implementing),

d - **Analysing** (by breaking the material into its parts and determining how these parts depends one from the other, and how they integrate to the whole structure),

e - **Evaluating** (by making judgments based on standards and criteria), and

f - **Creating** (the highest levels were interchanged).

- The set Q = {*i, a, b, c, d, e, f*} will be used in the following (*i* is from initial state of any learner).

Intuitionistic Fuzzy Human Learning -3

- During assessment, a collection of tasks are used corresponding to every level: Ta – to asses the recognizing or remembering process, Tb – to asses the understanding process, Tc – to asses the process of executing tasks, Td – to asses the analyzing abilities, Te – to assess the judgment process, and Tf to asses the power of creativity after learning the material.
- Let us denote by Σ the set of all tasks used during the assessment.
- An intuitionistic fuzzy pushdown context-free automaton can be constructed to model the incomplete progress from one level to another.

Intuitionistic Fuzzy Human Learning -4

- Let M = (Q, Σ, Γ, δ, I, Z0, F) be the generic model of pushdown automata, where Q is a nonempty set of states, Σ is the input alphabet, Γ is the pushdown alphabet (a stack based memory), δ gives the transition from one state to another along a Bloom trajectory, I is the initial state, Z0 is the initial knowledge (stored at the bottom of the stack), and F is the final state.
- For our developments Q plays the role of the universe set, that means I and F are intuitionistic fuzzy sets of Q. For simplicity we consider I as a crisp set, that means I = {*i*} as crisp set: mI(*i*) = 1, nI(*i*) = 0, and mI(x) = 0 (respective nI (x) = 1, for all x which are different from *i*). However, F is an intuitionistic fuzzy set in Q: F = {(q, m(q), n(q)), q in Q}.
- The transition function is defined as an intuitionistic fuzzy set in Q x ($\Sigma \cup \{\varepsilon\}$) x Γ x Q x Γ^*, with ε - the empty word (nothing to do), and Γ^* is the set of all sequences over Γ which can be stored and processed by stack oriented procedures.

Intuitionistic Fuzzy Human Learning – 5

- Ta = {Arrange_a, Define, Describe_a, Duplicate, Identify_a, Label, List, Match, Memorize, Name, Order, Outline_a, Recognise_a, Relate_a, Recall, Repeat, Reproduce, Select_a, State},
- Tb = {Classify, Convert, Describe_b, Defend_b, Discuss, Distinguish_b, Estimate_b, Explain_b, Express, Extend, Generalized, Give_example, Identify_b, Indicate, Infer_b, Locate, Paraphrase, Predict_b, Recognise_b, Rewrite_b, Review, Select_b, Summarize_b, Translate},
- Tc = {Apply, Change, Choose_c, Compute, Demonstrate, Discover, Dramatize, Employ, Illustrate_c, Interpret_c, Manipulate, Modify, Operate, Practice, Predict_c, Prepare_c, Produce, Relate_c, Schedule, Show, Sketch, Solve, Use, Write_c},
- Td = {Analyse, Appraise_d, Breakdown, Calculate, Categorise_d, Compare_d, Contrast_d, Criticise, Diagram, Differentiate, Discriminate_d, Distinguish_d, Examine, Experiment, Identify_d, Illustrate_d, Infer_d, Model, Outline_d, Point_out, Question, Relate_d, Select_d, Separate, Subdivide, Test},
- Te = {Appraise_e, Argue, Assess, Attach, Choose_e, Compare_e, Conclude, Contrast_e, Defend_e, Describe_e, Discriminate_e, Estimate_e, Evaluate, Explain_e, Judge, Justify, Interpret_e, Relate_e, Predict_e, Rate, Select_e, Summarize_e, Support, Value}, and
- Tf = {Arrange_f, Assemble, Categorise_f, Collect, Combine, Comply, Compose, Construct, Create, Design, Develop, Devise, Explain_f, Formulate, Generate, Plan, Prepare_f, Rearrange, Reconstruct, Relate_f, Reorganise_f, Revise, Rewrite_f, Set_up, Summarize_f, Synthesize, Tell, Write_f} .

Intuitionistic Fuzzy Human Learning - 6

- The above sets can be used to built, by union, sequences of tasks for assessment (belonging to Σ^*, where $\Sigma = Ta \cup Tb \cup \ldots \cup Tf$). A special set of symbols can be considered in order to store (in the pushdown memory) the history of the assessment evolution.
- The evolution of the assessment process can be illustrated by using an intuitionistic fuzzy binary relation $\Rightarrow$ defined according to the following rules:

R1) $m\Rightarrow((p, u, \alpha), (q, v, \beta)) = m\delta\,(p, \varepsilon, \mathit{head}(\alpha), q, \beta\backslash \mathit{tail}(\alpha))$ if $v = u$ and $\mathit{tail}(\alpha)\leq\beta$;

R2) $m\Rightarrow((p, u, \alpha), (q, v, \beta)) = m\delta\,(p, \mathit{head}(u), \mathit{head}(\alpha), q, \beta\backslash \mathit{tail}(\alpha))$ if $v = \mathit{tail}(u)$ and $\mathit{tail}(\alpha)\leq\beta$;

R3) $m\Rightarrow((p, u, \alpha), (q, v, \beta)) = 0$, otherwise;

R4) $n\Rightarrow((p, u, \alpha), (q, v, \beta)) = n\delta\,(p, \varepsilon, \mathit{head}(\alpha), q, \beta\backslash \mathit{tail}(\alpha))$ if $v = u$ and $\mathit{tail}(\alpha)\leq\beta$;

R5) $n\Rightarrow((p, u, \alpha), (q, v, \beta)) = n\delta\,(p, \mathit{head}(u), \mathit{head}(\alpha), q, \beta\backslash \mathit{tail}(\alpha))$ if $v = \mathit{tail}(u)$ and $\mathit{tail}(\alpha)\leq\beta$;

R6) $n\Rightarrow((p, u, \alpha), (q, v, \beta)) = 0$, otherwise,

where (p, u, α) and (q, v, β) belong to $Q \times \Sigma^* \times \Gamma^*$, and if u is nonempty, $u = x1x2\ldots xn$, then *head*(u) = x1, and *tail*(u) = x2...xn, n>1. Processing the assessment files is possible using the reflexive and transitive closure of this intuitionistic fuzzy binary relation.

- It is clear that given an assessment file and a learner (modelled by his/her intutitionistic fuzzy transition relation δ), this approach establishes for every state, starting from *i*, a pair of degrees that measure the intuitionistic fuzzy level of capability to do tasks related to the considered level. If necessary the researcher can use crispification methods under information loss acceptance.

On some intuitionistic fuzzy models ….

- References …

Previously published results:

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3. Albeanu G, Towards Intuitionistic Fuzzy Computational Models Of Learning, The 9th eLearning and Software for Education Conference - eLSE 2013, Bucharest, April 25-26, 2013.

- Questions ….
- …
- Thank you!